Name ANSWERS	Class

MATHS TEACHER HUB

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Algebraic fractions

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

10 Simplify $\dfrac{4(y+3)^3}{(y+3)^2}$ $= \dfrac{4(y+3)(y+3)(y+3)}{(y+3)(y+3)} = 4(y+3)$

$4y+12$

(1)

November 2019 – Paper 3H

(Total for Question 10 is 1 mark)

10 Simplify $\dfrac{x-1}{5(x-1)^2}$ $= \dfrac{(x-1)}{5(x-1)(x-1)} = \dfrac{1}{5(x-1)}$

$\dfrac{1}{5x-5}$

(1)

November 2018 – Paper 1H

(Total for Question 10 is 1 mark)

11 Solve $\frac{3x-2}{4} - \frac{2x+5}{3} = \frac{1-x}{6}$

$$\frac{3(3x-2)}{12} - \frac{4(2x+5)}{12} = \frac{2(1-x)}{12}$$

$$\frac{9x-6}{12} - \frac{8x+20}{12} = \frac{2-2x}{12}$$

$$9x-6-8x-20 = 2-2x$$

$$x-26 = 2-2x$$

$$3x = 28$$

$$x = \frac{28}{3}$$

$x =$ $\frac{28}{3}$ or $9\frac{1}{3}$

(Total for Question 11 is 4 marks)

12 Express $\frac{x}{x+2} + \frac{2x}{x-4}$ as a single fraction in its simplest form.

$$= \frac{x(x-4)}{(x+2)(x-4)} + \frac{2x(x+2)}{(x+2)(x-4)}$$

$$= \frac{x^2 - 4x}{(x+2)(x-4)} + \frac{2x^2 + 4x}{(x+2)(x-4)}$$

$$= \frac{3x^2}{(x+2)(x-4)}$$

$\frac{3x^2}{(x+2)(x-4)}$

(3)

November 2020 – Paper 3H

(Total for Question 12 is 3 marks)

12 (a) Write $\frac{4x^2 - 9}{6x + 9} \times \frac{2x}{x^2 - 3x}$ in the form $\frac{ax + b}{cx + d}$ where a, b, c and d are integers.

$$= \frac{(2x+3)(2x-3)}{3(2x+3)} \times \frac{2x}{x(x-3)}$$

$$= \frac{2x-3}{3} \times \frac{2x}{x(x-3)}$$

$$= \frac{2x(2x-3)}{3x(x-3)} = \frac{2(2x-3)}{3(x-3)}$$

$\frac{4x-6}{3x-9}$

(3)

(b) Express $\frac{3}{x+1} + \frac{1}{x-2} - \frac{4}{x}$ as a single fraction in its simplest form.

$$= \frac{3(x-2)(x)}{(x+1)(x-2)(x)} + \frac{1(x+1)(x)}{(x+1)(x-2)(x)} - \frac{4(x+1)(x-2)}{(x+1)(x-2)(x)}$$

$$= \frac{3x(x-2)}{(x+1)(x-2)(x)} + \frac{x(x+1)}{(x+1)(x-2)(x)} - \frac{4(x+1)(x-2)}{(x+1)(x-2)(x)}$$

$$= \frac{3x^2 - 6x}{(x+1)(x-2)(x)} + \frac{x^2 + x}{(x+1)(x-2)(x)} - \frac{4x^2 - 4x - 8}{(x+1)(x-2)(x)}$$

$$= \frac{-x + 8}{(x+1)(x-2)(x)}$$

$\frac{8-x}{(x+1)(x-2)(x)}$

(3)

(Total for Question 12 is 6 marks)

13 Show that $6 + \left[(x + 5) \div \dfrac{x^2 + 3x - 10}{x - 1} \right]$ simplifies to $\dfrac{ax - b}{cx - d}$ where a, b, c and d are integers.

$= 6 + \left[(x+5) \div \dfrac{(x+5)(x-2)}{(x-1)} \right]$

$= 6 + \left[\cancel{(x+5)} \times \dfrac{(x-1)}{\cancel{(x+5)}(x-2)} \right]$

$= 6 + \left[\dfrac{(x-1)}{(x-2)} \right]$

$= \dfrac{6(x-2)}{(x-2)} + \dfrac{(x-1)}{(x-2)}$

$= \dfrac{6x - 12 + x - 1}{(x-2)}$

$= \dfrac{7x - 13}{(x-2)}$

(Total for Question 13 is 4 marks)

13 (a) Write $\frac{5}{x+1} + \frac{2}{3x}$ as a single fraction in its simplest form.

$$= \frac{5(3x)}{(x+1)(3x)} + \frac{2(x+1)}{(x+1)(3x)}$$

$$= \frac{15x}{(x+1)(3x)} + \frac{2x+2}{(x+1)(3x)}$$

$$= \frac{17x+2}{(x+1)(3x)}$$

$$\frac{17x+2}{(x+1)(3x)}$$

(2)

November 2019 – Paper 1H

(Total for Question 13 is 2 marks)

14 The ratio $(y + x) : (y - x)$ is equivalent to $k : 1$

Show that $y = \frac{x(k+1)}{k-1}$

$$\frac{y+x}{y-x} = \frac{k}{1}$$

$$y + x = k(y - x)$$

$$y + x = ky - kx$$

$$y + x + kx = ky$$

$$x + kx = ky - y$$

$$x(1+k) = y(k-1)$$

$$y = \frac{x(1+k)}{(k-1)}$$

November 2017 – Paper 1H

(Total for Question 14 is 3 marks)

14 Simplify $\dfrac{x^2 - 16}{2x^2 - 5x - 12}$

$= \dfrac{(x+4)(x-4)}{(2x+3)(x-4)}$

$= \dfrac{(x+4)}{(2x+3)}$

$\dfrac{x+4}{2x+3}$

(3)

June 2017 – Paper 3H

(Total for Question 14 is 3 marks)

14 Solve $\dfrac{x+2}{3x} + \dfrac{x-2}{2x} = 3$

$\dfrac{2(x+2)}{6x} + \dfrac{3(x-2)}{6x} = 3$

$\dfrac{2x+4}{6x} + \dfrac{3x-6}{6x} = 3$

$\dfrac{5x-2}{6x} = 3$

$5x - 2 = 18x$

$x =$ $\dfrac{-2}{13}$

Specimen 1 – Paper 1H

(Total for Question 14 is 3 marks)

$-2 = 13x$

$\dfrac{-2}{13} = x$

14 Show that $\dfrac{x^2 - x - 6}{2x^2 - 5x - 3}$ can be written in the form $\dfrac{ax + b}{cx + d}$ where a, b, c and d are integers.

$$= \frac{(x-3)(x+2)}{(2x+1)(x-3)}$$

$$= \frac{(x+2)}{(2x+1)}$$

$$= \frac{x+2}{2x+1}$$

(Total for Question 14 is 3 marks)

14 Write

$$4 - \left[(x+3) \div \frac{x^2 + 5x + 6}{x - 2} \right]$$

as a single fraction in its simplest form.
You must show your working.

$= 4 - \left[(x+3) \div \dfrac{(x+3)(x+2)}{(x-2)} \right]$

$= 4 - \left[(x+3) \times \dfrac{(x-2)}{(x+3)(x+2)} \right]$

$= 4 - \left[\dfrac{(x-2)}{(x+2)} \right]$

$= \dfrac{4(x+2)}{(x+2)} - \dfrac{(x-2)}{(x+2)} = \dfrac{4x+8-x+2}{(x+2)} = \dfrac{3x+10}{x+2}$

(Total for Question 14 is 4 marks)

15 (a) Simplify fully $\dfrac{(a-3)^2}{5(a-3)}$

$= \dfrac{(a-3)(a-3)}{5(a-3)}$

$= \dfrac{a-3}{5}$

$\dfrac{a-3}{5}$

(1)

(b) Simplify fully $\dfrac{4-x^2}{x^2+3x} \div \dfrac{x+2}{x+3}$

$= \dfrac{(2+x)(2-x)}{x(x+3)} \div \dfrac{(x+2)}{(x+3)}$

$= \dfrac{(2+x)(2-x)}{x(x+3)} \times \dfrac{(x+3)}{(x+2)}$

$= \dfrac{2-x}{x}$

$\dfrac{2-x}{x}$

(3)

(Total for Question 15 is 4 marks)

15 Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a, b and c are integers.

$$\frac{5(4x+3)}{10x} + \frac{3(2x)}{10x}$$

$$= \frac{20x+15}{10x} + \frac{6x}{10x}$$

$$= \frac{26x+15}{10x}$$

(Total for Question 15 is 3 marks)

15 Show that $\frac{2x^2 - 3x - 5}{x^2 + 6x + 5}$ can be written in the form $\frac{ax + b}{cx + d}$ where a, b, c and d are integers.

$$= \frac{(2x-5)(x+1)}{(x+5)(x+1)}$$

$$= \frac{(2x-5)}{(x+5)}$$

$$= \frac{2x-5}{x+5}$$

Specimen 1 – Paper 1H

(Total for Question 15 is 3 marks)

15 Show that $\frac{a}{b+1} - \frac{a}{(b+1)^2}$ can be written as $\frac{ab}{(b+1)^2}$

$$= \frac{a(b+1)}{(b+1)^2} - \frac{a}{(b+1)^2}$$

$$= \frac{ab+a}{(b+1)^2} - \frac{a}{(b+1)^2} \qquad = \frac{ab}{(b+1)^2}$$

Specimen 2 – Paper 3H

(Total for Question 15 is 2 marks)

16 Show that $\dfrac{1}{6x^2 + 7x - 5} \div \dfrac{1}{4x^2 - 1}$ simplifies to $\dfrac{ax + b}{cx + d}$ where a, b, c and d are integers.

$$= \frac{1}{(3x+5)(2x-1)} \div \frac{1}{(2x+1)(2x-1)}$$

$$= \frac{1}{(3x+5)(2x-1)} \times \frac{(2x+1)(2x-1)}{1}$$

$$= \frac{2x+1}{3x+5}$$

$\dfrac{2x+1}{3x+5}$

Sample 1 – Paper 2H

(Total for Question 16 is 3 marks)

17 Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

$$\frac{x^2}{3x+5} = \frac{1}{2}$$

$$2x^2 = 3x + 5$$

$$2x^2 - 3x - 5 = 0$$

$$(2x - 5)(x + 1) = 0$$

$$x = \frac{5}{2} \qquad x = -1$$

$x = 2.5$ or $x = -1$

(Total for Question 17 is 4 marks)

17 Simplify fully $\dfrac{3x^2 - 8x - 3}{2x^2 - 6x}$

$$= \frac{(3x+3)(x-3)}{2x(x-3)}$$

$$= \frac{(3x+3)}{2x}$$

$$\frac{3x+3}{2x}$$

June 2018 – Paper 1H

(Total for Question 17 is 3 marks)

19 Solve $\frac{1}{2x-1}+\frac{3}{x-1}=1$

Give your answer in the form $\frac{p\pm\sqrt{q}}{2}$ where p and q are integers.

$$\frac{x-1}{(2x-1)(x-1)}+\frac{3(2x-1)}{(2x-1)(x-1)}=1$$

$$\frac{x-1}{(2x-1)(x-1)}+\frac{6x-3}{(2x-1)(x-1)}=1$$

$$\frac{7x-4}{(2x-1)(x-1)}=1$$

$$7x-4=(2x-1)(x-1)$$

$$7x-4=2x^2-3x+1$$

$$0=2x^2-10x+5$$

$$a \quad b \quad c$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-10)+\sqrt{(-10)^2-4\times2\times5}}{4}$$

$$x=\frac{10+\sqrt{100-40}}{4}$$

$$x=\frac{10+\sqrt{60}}{4}$$

$$x=\frac{10+2\sqrt{15}}{4}$$

$$x=\frac{10\pm2\sqrt{15}}{4}$$

(Total for Question 19 is 4 marks)

19 Show that $\frac{3x}{x+2} - \frac{2x+1}{x-2} - 1$ can be written in the form $\frac{ax+b}{x^2-4}$ where a and b are integers.

$$\frac{3x(x-2)}{(x+2)(x-2)} - \frac{(2x+1)(x+2)}{(x+2)(x-2)} - \frac{(x+2)(x-2)}{(x+2)(x-2)}$$

$$= \frac{3x^2-6x}{(x+2)(x-2)} - \frac{2x^2+5x+2}{(x+2)(x-2)} - \frac{x^2-4}{(x+2)(x-2)}$$

$$= \frac{-11x+2}{(x+2)(x-2)}$$

$$= \frac{-11x+2}{x^2-4}$$

June 2022 – Paper 3H

(Total for Question 19 is 4 marks)

19 Solve $\frac{1}{x} - \frac{1}{x+1} = 4$

Give your answer in the form $a \pm b\sqrt{2}$ where a and b are fractions.

$$\frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} = 4$$

$$\frac{1}{x(x+1)} = 4$$

$$\frac{1}{x^2+x} = 4$$

$$1 = 4x^2 + 4x$$

$$0 = 4x^2 + 4x - 1$$

$a \quad b \quad c$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2-4\times4\times(-1)}}{8}$$

$$x = \frac{-4 \pm \sqrt{32}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{2}}{8}$$

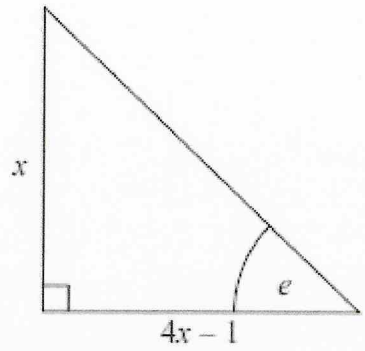
$$x = -\frac{4}{8} \pm \frac{4\sqrt{2}}{8}$$

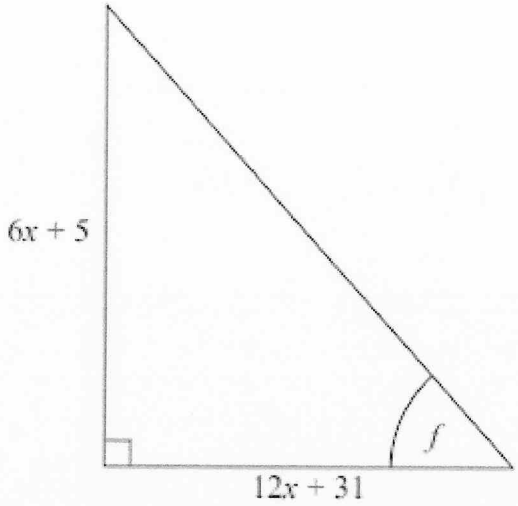
$$x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{2}$$

$$x = -0.5 \pm 0.5\sqrt{2}$$

(Total for Question 19 is 5 marks)

19 Here are two right-angled triangles.





Given that

$$\tan e = \tan f$$

find the value of x.

You must show all your working.

$$\frac{x}{4x-1} = \frac{6x+5}{12x+31}$$

$$x(12x+31) = (6x+5)(4x-1)$$

$$12x^2 + 31x = 24x^2 + 14x - 5$$

$$0 = 12x^2 - 17x - 5$$

$a = 12$, $b = -17$, $c = -5$

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \times 12 \times (-5)}}{24}$$

$$x = \frac{5}{3} \quad \text{or} \quad -\frac{1}{4}$$

$x = \frac{5}{3}$

June 2018 – Paper 3H **(Total for Question 19 is 5 marks)**

19 $2 - \frac{x+2}{x-3} - \frac{x-6}{x+3}$ can be written as a single fraction in the form $\frac{ax+b}{x^2-9}$ where a and b are integers.

Work out the value of a and the value of b.

$$= \frac{2(x-3)(x+3)}{(x-3)(x+3)} - \frac{(x+2)(x+3)}{(x-3)(x+3)} - \frac{(x-6)(x-3)}{(x-3)(x+3)}$$

$$= \frac{2(x^2-9)}{x^2-9} - \frac{x^2+5x+6}{x^2-9} - \frac{x^2-9x+18}{x^2-9}$$

$$= \frac{2x^2-18}{x^2-9} - \frac{x^2+5x+6}{x^2-9} - \frac{x^2-9x+18}{x^2-9}$$

$$= \frac{4x-42}{x^2-9}$$

$a =$ 4

$b =$ −42

June 2017 – Paper 2H **(Total for Question 19 is 4 marks)**

20 Write

$$\frac{14}{3x-21}+\left[(x+4)\div\frac{2x^2-6x-56}{2x+3}\right]$$ in the form $\frac{ax+b}{cx+d}$ where a, b, c and d are integers.

$2x^2-6x-56 = 2(x^2-3x-28)$

$$= \frac{14}{3x-21}+\left[(x+4)\div\frac{2(x+4)(x-7)}{2x+3}\right]$$

$$= \frac{14}{3x-21}+\left[(x+4)\times\frac{2x+3}{2(x+4)(x-7)}\right]$$

$$= \frac{14}{3x-21}+\left[\frac{2x+3}{2(x-7)}\right]$$

$$= \frac{14}{3(x-7)}+\frac{2x+3}{2(x-7)}$$

×2 ×3

$$= \frac{28}{6(x-7)}+\frac{6x+9}{6(x-7)}$$

$$= \frac{6x+37}{6(x-7)}$$

$\frac{6x+37}{6x-42}$

(Total for Question 20 is 4 marks)

20 Show that $\dfrac{3x+6}{x^2-3x-10} \div \dfrac{x+5}{x^3-25x}$ simplifies to ax where a is an integer.

$$= \frac{3(x+2)}{(x+2)(x-5)} \div \frac{(x+5)}{x(x^2-25)}$$

$$= \frac{3(x+2)}{(x+2)(x-5)} \times \frac{x(x+5)(x-5)}{(x+5)} = 3x$$

(Total for Question 20 is 4 marks)

21 Solve $\frac{1}{x+4}+\frac{3}{2-2x}=1$

$$\frac{2-2x}{(x+4)(2-2x)}+\frac{3(x+4)}{(x+4)(2-2x)}=1$$

$$\frac{2-2x}{(x+4)(2-2x)}+\frac{3x+12}{(x+4)(2-2x)}=1$$

$$\frac{14+x}{(x+4)(2-2x)}=1$$

$$14+x=(x+4)(2-2x)$$

$$14+x=-2x^2-6x+8$$

$$2x^2+7x+6=0$$

$$(2x+3)(x+2)=0$$

$2x+3=0$

$x=\frac{-3}{2}$

$x+2=0$

$x=-2$

$x=-1.5$ or -2

(Total for Question 21 is 4 marks)

22 Show that $\frac{6x^3}{(9x^2 - 144)} \div \frac{2x^4}{3(x - 4)}$ can be written in the form $\frac{1}{x(x + r)}$ where r is an integer.

$$= \frac{6x^3}{(3x+12)(3x-12)} \div \frac{2x^4}{3(x-4)}$$

$$= \frac{6x^3}{3(x+4) \times 3(x-4)} \times \frac{3(x-4)}{2x^4}$$

$$= \frac{6x^3}{9(x+4)(x-4)} \times \frac{3(x-4)}{2x^4}$$

$$= \frac{18x^3}{18x^4(x+4)}$$

$$= \frac{1}{x(x+4)}$$

(Total for Question 22 is 3 marks)

22 $\frac{2x+3}{x-5}+\frac{x-4}{x+5}-3$ can be written in the form $\frac{ax+b}{x^2-25}$ where a and b are integers.

Work out the value of a and the value of b.
You must show all your working.

$$=\frac{(2x+3)(x+5)}{(x-5)(x+5)}+\frac{(x-4)(x-5)}{(x-5)(x+5)}-\frac{3(x-5)(x+5)}{(x-5)(x+5)}$$

$$=\frac{2x^2+13x+15}{x^2-25}+\frac{x^2-9x+20}{x^2-25}-\frac{3x^2-75}{x^2-25}$$

$$=\frac{4x+110}{x^2-25}$$

$a =$ 4

$b =$ 110

(Total for Question 22 is 3 marks)

22 Show that $\dfrac{7x-14}{x^2+4x-12} \div \dfrac{x-6}{x^3-36x}$ simplifies to ax where a is an integer.

$$= \frac{7(x-2)}{(x+6)(x-2)} \div \frac{(x-6)}{x(x^2-36)}$$

$$= \frac{7(x-2)}{(x+6)(x-2)} \div \frac{(x-6)}{x(x+6)(x-6)}$$

$$= \frac{7\cancel{(x-2)}}{\cancel{(x+6)(x-2)}} \times \frac{x\cancel{(x+6)}\cancel{(x-6)}}{\cancel{(x-6)}}$$

$$= 7x$$

(Total for Question 22 is 4 marks)

23 Given that $\frac{2x^2 + y^2}{4x^2 - y^2} = \frac{43}{11}$ where $x > 0$ and $y > 0$

find, in its simplest form, the ratio $x : y$

$11(2x^2+y^2) = 43(4x^2-y^2)$

$22x^2 + 11y^2 = 172x^2 - 43y^2$

$54y^2 = 150x^2$

$\sqrt{54}\sqrt{y^2} = \sqrt{150}\sqrt{x^2}$

$3\sqrt{6}y = 5\sqrt{6}x$

$3y = 5x$

$x : y$

$3 : 5$

$3 : 5$

(Total for Question 23 is 4 marks)